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Bubbly Recessions†

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We develop a tractable bubbles model with financial friction and downward wage rigidity. Competitive speculation in risky bubbles can result in excessive investment booms that precede inefficient busts, where post-bubble aggregate economic activities collapse below the pre-bubble trend. Risky bubbles can reduce ex ante social welfare, and leaning-against-the-bubble policies that balance the boom-bust trade-off can be warranted. We further show that the collapse of a bubble can push the economy into a "secular stagnation" equilibrium, where the zero lower bound and the nominal wage rigidity constraint bind, leading to a persistent recession, such as the Japanese "lost decades." (*JEL* E22, E24, E32, E44, L26)

In the recent decades, many countries in the world, including Japan, the United States, and several European economies, have experienced episodes of rapid speculative booms and busts in asset prices followed by declines in economic activities, and, in some cases, persistent recessions. More generally, throughout history, the collapse of large asset and credit booms tend to precede recessions and crises (e.g., Kindleberger and Aliber 2005, Jordà et al. 2015). These experiences have led policymakers to be increasingly aware of the potential risks of asset price bubbles, leading to discussions of macroprudential regulations such as "leaning-against-the-wind" policies—preventive measures to curb the booms in asset prices in order to mitigate the eventual busts.

However, despite the recent developments in the macroeconomic literature on asset bubbles, relatively little theoretical framework has analyzed the potential efficiency trade-off between the booms and busts of risky bubbly episodes and whether preventive policies are warranted. In particular, in most rational bubble models—the

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workhorse models to study the macroeconomic effects of bubbles in general equilibrium—private agents correctly perceive the risk of speculating in a bubbly asset, and bubbles generally improve the efficiency of the financial system (Barlevy 2018; Miao 2014; Martin and Ventura 2018).

In this paper, we develop a tractable general equilibrium model to address the question of when and how risky rational bubbles can lead to inefficiencies and evaluate the welfare trade-off. We focus on the combination of financial friction and downward wage rigidities during bubbly episodes. We posit an economy where entrepreneurial agents with heterogeneous productivity accumulate capital and face financial friction that constrains their ability to borrow from each other. If the credit and capital markets cannot satisfy the demand for savings, speculative bubbles may arise. A rational bubble is an asset that is traded above its fundamental value; an agent purchases the overvalued asset because he or she expects to be able to sell it later. We assume bubbles are stochastic in the sense that in each period the price of the bubbly asset can collapse to the fundamental value with an exogenous probability (e.g., Blanchard and Watson 1982; Weil 1987).

The possibility of trading the bubbly asset facilitates the reallocation of resources across time, because the bubbly asset can act as a savings vehicle. Trading also facilitates reallocation across agents, because the bubbly asset increases entrepreneurs' net worth and hence their ability to borrow. Thus, the boom in the price of a bubbly asset leads to a boom in entrepreneurial net worth, credit, investment, output, wages, and consumption. When the boom finally turns into a bust, the economy simply converges back to the pre-bubble economy. Therefore, with financial friction alone, the model, so far, implies that speculative bubbles help to crowd in productive investment and improve the overall efficiency of the economy, as implied in most existing expansionary bubble frameworks (e.g., Hirano et al. 2015; Miao and Wang 2018).

However, the implications change with downward wage rigidities. When an expansionary bubble collapses, the net worth of entrepreneurial agents also collapses, leading to contractions in credit, investment, and labor demand. In a flexible labor market, wages will fall to clear the labor market. However, we assume that (real) wages are downwardly rigid. Then there will be rationing in the labor market, resulting in involuntary unemployment. An increase in unemployment can in turn lead to an endogenous and protracted recession by eroding the intertemporal allocation of resources. The drop in employment reduces the return to capital investment, which then lowers entrepreneurs' net worth. This further leads to a contraction in capital investment, since entrepreneurs' ability to borrow and invest depends critically on their net worth. Therefore, the future capital stock will decline, causing further downward pressure on labor demand and wages, thus reducing future capital accumulation. The vicious cycle continues until the capital stock has fallen enough, often *undershooting* the bubbleless steady-state level.

In short, our theory identifies the booms and busts of speculative bubbly episodes as an important source of shocks that can potentially trigger a deep and persistent recession, such as the lost decades in Japan or the Great Depression and Great Recession in the United States. We further show that when the bubble is sufficiently risky and the labor market is sufficiently rigid, society's welfare can be better off without bubbles. Our model thus provides a step toward bridging the views of policymakers and theoretical models of bubbles (Barlevy 2018). In particular, our theory naturally implies that a "leaning-against-the-bubble" type of macroprudential policy intervention is warranted for excessively large bubbles. The source of inefficiencies is a form of "bubbly pecuniary externality," as individual investors do not internalize the effect of their portfolio choices in driving a large bubbly boom, which will lead to a large bust.

Finally, we extend the real model to an environment where *nominal* wages are downwardly rigid (Schmitt-Grohé and Uribe 2016, 2017) and the central bank sets the nominal interest rate according to a Taylor rule that is subject to the zero lower bound (ZLB). We then show that the collapse of a large expansionary bubble triggers a sharp drop in the real interest rate, pushing the nominal interest rate against the ZLB. Intuitively, by crowding in capital investment, the bubble leads to an investment boom. Thus, after the bubble collapses, the economy experiences an "investment overhang," as it has too much capital relative to the bubbleless steady state. The high capital stock implies a low marginal product of capital and a low real interest rate. The collapse of a sufficiently large bubble can thus push the real interest rate so low that the ZLB binds. We show that under certain conditions, the post-bubble economy may fall into a "secular stagnation" steady state, where employment and investment are persistently and inefficiently low and inflation is below target. A vicious cycle can arise from the interaction between (i) a low interest rate environment, which constrains the monetary authority from raising inflation, exacerbating the nominal wage rigidity and unemployment problem and (ii) inefficient unemployment that lowers the marginal product of capital, which in turn lowers the interest rates. In the absence of other shocks, this cycle can keep the economy in a persistent slump.

Related Literature.—By showing that collapse of risky bubbles can trigger inefficient recessions, our paper is related to several strands of the bubble literature. A large number of papers emphasize the positive aspect of bubbles in reducing dynamic inefficiencies (e.g., Samuelson 1958; Diamond 1965; Tirole 1985) or reducing allocative inefficiencies (e.g., Farhi and Tirole 2012; Miao and Wang 2012, 2018; Martin and Ventura 2012; Graczyk and Phan 2016; Ikeda and Phan 2019). Other papers emphasize potential ex ante inefficiencies of speculative bubbles in diverting resources away from productive investment (e.g., Saint-Paul 1992; Grossman and Yanagawa 1993; King and Ferguson 1993; Hirano et al. 2015), or generating excessive investment in certain sectors (e.g., Cahuc and Challe 2012; Miao, Wang, and Xu 2015b), excessive volatility (Caballero and Krishnamurthy 2006; Ikeda and Phan 2016), or excessive default (Kocherlakota 2009; Barlevy 2014; Bengui and Phan 2018).

In highlighting the adverse effects of collapsing bubbles in an environment with wage rigidity, our paper complements the work by Miao and Wang (2015) who show that in an environment with bubbles in bank values, collapsing bubbles can trigger a sharp contraction in bank lending and push economic activities below the pre-bubble trend.

Our framework is related to a growing literature on monetary models with bubbles, including the infinite-lived agent models of Dong, Miao, and Wang (2018); Ikeda (2018); and Hirano et al. (2017), and the overlapping generation models of Galí

(2014, 2016), Asriyan et al. (2016), and Hanson and Phan (2017). Our paper complements this literature by showing that the combination of an expansionary bubble and downward nominal wage rigidity can cause a post-bubble liquidity trap.

Our paper is also related to a large macroeconomic literature that investigates the causes of liquidity traps, notably deleveraging shocks (e.g., Eggertsson and Krugman 2012; Korinek and Simsek 2016; Buera and Nicolini 2017) and shocks to inflation expectations (e.g., Schmitt-Grohé and Uribe 2017) or idiosyncratic risk (e.g., Christiano, Motto, and Rostagno 2014; Acharya and Dogra 2018). The buildup of an expansionary bubble in our model provides a microfoundation to the investment overhang in Simsek, Shleifer, and Rognlie (2014), and its collapse provides a microfoundation for the deleverage shocks in Eggertsson and Krugman (2012).

Finally, by analyzing macroprudential policies on speculative bubbles, our paper complements Biljanovska et al. (2019) and contributes to the literature on macroprudential policies (e.g., Lorenzoni 2008; Olivier and Korinek 2010; He and Krishnamurthy 2012; Bianchi 2011; Eberly and Krishnamurthy 2014; Farhi and Werning 2016; Bianchi and Mendoza 2018).

I. Model

Consider an economy with two types of goods: a perishable consumption good and a capital good. There are two types of agents, called entrepreneurs and workers, each with constant unit population. They have identical preferences,

$$E_0\left(\sum_{t=0}^{\infty}\beta^t u(c_t)\right),$$

where the period utility function is $u(c) = \log(c)$, $\beta \in (0,1)$ is the subjective discount factor, and $E_0(\cdot)$ is the expected value conditional on information in period 0.

A. *Entrepreneurs*

Entrepreneurs are the only producers of the capital good. They rent the capital produced to firms through a competitive capital rental market. They face idiosyncratic productivity shocks: in each period, an entrepreneur receives a random productivity shock a, where a is independently and identically distributed (i.i.d.) according to a continuous distribution with a cumulative distribution function (CDF) denoted by F.[1] For tractability, we assume that the distribution is Pareto over $[1,\infty)$ with shape parameter σ. In order for the distribution to have a finite mean, we assume

$$\sigma > 1.$$

[1] As is well-known among models with heterogeneous productivity shocks (e.g., Carlstrom and Fuerst 1997; Bernanke, Gertler, and Gilchrist 1999; Kocherlakota 2009; Liu and Wang 2014), the i.i.d. assumption helps keep the model tractable. A model with persistent idiosyncratic shocks can only be solved numerically.

We denote the set of entrepreneurs by $J \equiv [0,1]$. After knowing the idiosyncratic productivity shock, an entrepreneur $j \in J$ produces the capital good according to the following technology:

$$k_{t+1}^j = a_t^j I_t^j,$$

where I_t^j is the investment in units of the consumption good in period t, k_{t+1}^j is the amount of the capital good produced in the subsequent period, and a_t^j is the productivity of the entrepreneur. For tractability, we assume capital depreciates completely after being used in each period (we relax this assumption in the online Appendix and in the numerical analysis).

Following the bubbles literature (e.g., Tirole 1985), we assume there is a durable and perfectly divisible financial asset in fixed unit supply that does not generate any dividend. In an equilibrium, which we call the bubbleless equilibrium, the asset will be priced at its fundamental value of zero. In some other equilibria, which we call bubbly equilibria, the asset will have a positive price. Let b_t^j denote a share of the asset held by entrepreneur j and p_t^b be the price per unit of the asset. Then the entrepreneur's flow budget constraint is written as

$$c_t^j + I_t^j + p_t^b b_t^j = R_t^k k_t^j + d_t^j - R_{t-1,t} d_{t-1}^j + p_t^b b_{t-1}^j, \tag{1}$$

where $R_{t-1,t}$ is the gross interest rate between $t-1$ and t, d_t^i is the amount of net borrowing in period t, and R_t^k is the rental rate of capital in t. The left-hand side of this budget constraint consists of expenditure on consumption, capital investment, and the purchase of financial asset. The right-hand side is the available funds at date t, which consists of the return from capital investment in the previous period, new net borrowing minus the net debt repayment, and the return from selling the financial asset. Agents cannot invest a negative amount in the capital stock or the asset, i.e.,[2]

$$I_t^j, b_t^j \geq 0, \quad \forall t. \tag{2}$$

The entrepreneur's net worth at the beginning of period t is

$$e_t^j \equiv R_t^k a_{t-1}^j I_{t-1}^j - R_{t-1,t} d_{t-1}^j + p_t^b b_{t-1}^j. \tag{3}$$

Following Moll (2014), we assume that due to financial friction, entrepreneurs can finance at most a fraction $\theta \in [0,1]$ of their capital investment with external credit:

$$d_t^j \leq \theta I_t^j. \tag{4}$$

In equilibrium, this will imply that an entrepreneur's leverage is constrained by

$$\frac{d_t^j}{e_t^j} \leq \beta \frac{\theta}{1-\theta}.$$

[2] As otherwise, the ability to short sell would let agents borrow and bypass credit constraint (4).

In general, a larger θ can be interpreted as representing an environment with less financial friction. This type of simple credit constraint has been used extensively in recent general equilibrium models with heterogeneous agents (e.g., Banerjee and Moll 2010; Buera and Shin 2013), and it allows us to get analytical solutions to the model.

The optimization problem of each entrepreneur j is as follows. In each period after knowing her productivity shock a_t^j, the entrepreneur chooses consumption c_t^j, capital investment I_t^j, net debt position d_t^j (where a negative d_t^j means lending), and net asset purchase $b_t^j - b_{t-1}^j$. Her objective is to maximize the lifetime expected utility $E_t(\sum_{s\geq 0} \beta^s \log c_{t+s}^j)$, subject to budget constraint (1), nonnegativity constraint (2), and credit constraint (4).

B. *Workers*

Workers do not have access to capital production technologies. For simplicity, we assume workers are hand to mouth, i.e.,

$$c_t^w = w_t l_t, \tag{5}$$

where w_t is the wage rate and l_t is the employment level per worker.[3]

C. *Firms*

In each period, there is a continuum of competitive firms that produce the consumption good from hiring labor from workers and renting capital from entrepreneurs. Their production function is standard:

$$y_t^i = \left(k_t^i\right)^{\alpha} \left(l_t^i\right)^{1-\alpha}, \; 0 < \alpha < 1,$$

where k_t^i and l_t^i are capital and labor inputs of a representative firm i. Competitive factor prices are given by the marginal products of capital and labor:

$$R_t^k = \alpha \left(\frac{L_t}{K_t}\right)^{1-\alpha} \tag{6}$$

$$w_t = (1-\alpha)\left(\frac{K_t}{L_t}\right)^{\alpha}, \tag{7}$$

where K_t and L_t are the aggregate capital stock and employment.

[3] Alternatively, we can assume workers cannot borrow against their future labor income. Thus, the optimization problem of workers is to maximize lifetime utility $E_0(\sum_{t=0}^{\infty} \beta^t \ln c_t^w)$ subject to

$$c_t^w + p_t^b b_t^w = w_t l_t + d_t^w - R_t d_{t-1}^w + p_t^b b_{t-1}^w,$$

and $d_t^w \leq 0$ and $b_t^w \geq 0$. In equilibrium, it can be shown that workers will be effectively hand to mouth, i.e., $c_t^w = w_t l_t$. Intuitively, due to financial friction, the interest rate (and the returns from bubble speculation) will be too low relative to the discount factor, and thereby it will be suboptimal for workers to save or to buy the bubbly asset (see Hirano. Inaba, and Yanagawa 2015 for more details).

D. *Downward Wage Rigidity (DWR)*

We assume that real wages are downwardly rigid:

$$w_t \geq \gamma w_{t-1}, \quad \forall t \geq 1, \tag{8}$$

where $\gamma \in [0,1]$ is a constant parameter that governs the degree of rigidity. The condition states that the real wage cannot fall below a certain fraction of the real wage in the last period.[4]

The presence of rigid wages implies that the labor market does not necessarily clear. In each period, even though each worker inelastically supplies one unit of labor, the realized employment L_t per worker in equilibrium is determined by two conditions: a feasibility constraint:

$$L_t \leq 1, \tag{9}$$

and a complementary-slackness condition:

$$(1 - L_t)(w_t - \gamma w_{t-1}) = 0. \tag{10}$$

These equations state that involuntary unemployment ($L_t < 1$) must be accompanied by a binding wage rigidity (8). Conversely, when (8) is slack, the economy must be in full employment ($L_t = 1$). For simplicity, we also assume that in the initial period $t = 0$, the legacy wage w_{-1} is sufficiently small, so that the labor market clears in $t = 0$.

E. *Equilibrium*

DEFINITION 1: *Given initial* $k_0^j = K_0$, $d_0^j = 0$, $b_0^j = 1$, p_0^b, *a competitive equilibrium consists of prices* $\{w_t, R_t^k, R_{t,t+1}, p_t^b\}_{t \geq 0}$ *and quantities* $\{\{I_t^j, k_{t+1}^j, c_t^j\}_{j \in J}, c_t^w, K_{t+1}, L_t\}_{t \geq 0}$ *such that:*

- *Entrepreneurs and firms optimize.*
- *The consumption of a representative worker is given by* (5).
- *The credit market clears*: $\int_0^1 d_t^j \, dj = 0$.
- *The asset market clears*: $\int_0^1 b_t^j \, dj = 1$.
- *The consumption good market clears*: $\int_0^1 (c_t^j + I_t^j) \, dj + c_t^w = K_t^\alpha L_t^{1-\alpha}$.
- *The capital market clears*: $K_t = \int_0^1 k_t^j \, dj$.
- *Labor market conditions* (8), (9), *and* (10) *hold.*

As usual, a steady state is an equilibrium where quantities, prices (in units of the consumption good), and inflation are time invariant.

[4] For empirical evidence of real wage rigidity, see, e.g., Holden and Wulfsberg (2009) and Babecký et al. (2010).

II. Bubbleless Benchmark

Let us first analyze the bubbleless equilibrium, where the price of the financial asset is equal to its fundamental value of zero, i.e., $p_t^b = 0$ for all t. Details of the derivations are relegated to the online Appendix.

A. *Optimal Decisions of Individual Entrepreneurs*

In each period t, given the realization of her productivity shock a_t^j, each entrepreneur j chooses c_t^j, I_t^j, and d_t^j. Since the period utility function is logarithmic, the optimal action for the entrepreneur is to consume a fraction $1 - \beta$ of her net worth e_t^j:

$$c_t^j = (1 - \beta) e_t^j, \tag{11}$$

and invest/save the remaining fraction β:

$$I_t^j + \left(-d_t^j\right) = \beta e_t^j. \tag{12}$$

In the bubbleless benchmark, net worth (as defined in (3)) is simply capital income minus net debt repayment:

$$e_t^j = R_t^k a_{t-1}^j I_{t-1}^j - R_{t-1,t} d_{t-1}^j.$$

Both the savings options of investing in capital and lending in the credit market are riskless. Hence, the entrepreneur will simply choose the option that offers the highest rate of return. Lending yields a rate of return $R_{t,t+1}$, which is the same for all entrepreneurs. Capital investment yields a rate of return $a_t^j R_{t+1}^k$, which varies according to each entrepreneur's productivity a_t^j. Hence, in equilibrium, there is a *cutoff productivity threshold* $\bar{a}_t$ in each period such that all entrepreneurs with $a_t^j < \bar{a}_t$ will only lend and not invest in capital (i.e., the constraint $I_t^j \geq 0$ binds), while those with $a_t^j > \bar{a}_t$ will only invest in capital and borrow as much as possible (i.e., credit constraint (4) binds). Entrepreneurs with $a_t^j = \bar{a}_t$ (the "marginal investors") will be indifferent between lending and investing in capital, and their d_t^j and I_t^j are indeterminate. The indifference condition yields a mapping between the interest rate and the marginal product of capital:

$$R_{t,t+1} = \bar{a}_t R_{t+1}^k. \tag{13}$$

In summary, entrepreneurs' leverage ratio is given by[5]

$$\frac{d_t^j}{e_t^j} = \begin{cases} -\beta & \text{if } a_t^j < \bar{a}_t \\ \beta \dfrac{\theta}{1-\theta} & \text{if } a_t^j > \bar{a}_t \end{cases}. \tag{14}$$

[5] We ignore the indeterminate case of $a_t^j = \bar{a}_t$, as it happens with probability zero.

Their capital investment is given by

$$I_t^j = \begin{cases} 0 & \text{if } a_t^j < \bar{a}_t \\ \frac{\beta}{1-\theta} e_t^j & \text{if } a_t^j > \bar{a}_t \end{cases}, \tag{15}$$

and the amount of capital produced by each entrepreneur in $t+1$ is given by

$$k_{t+1}^j = \begin{cases} 0 & \text{if } a_t^j < \bar{a}_t \\ \frac{\beta a_t^j}{1-\theta} e_t^j & \text{if } a_t^j > \bar{a}_t \end{cases}. \tag{16}$$

B. *Aggregation*

Given the decisions of individual entrepreneurs, we can characterize the aggregate equilibrium dynamics. The aggregate net worth of entrepreneurs is equal to the aggregate capital income:

$$e_t = \int_0^1 e_t^j dj = \alpha K_t^\alpha L_t^{1-\alpha}. \tag{17}$$

The cutoff threshold $\bar{a}_t$ is determined by the credit market clearing condition $\int_0^1 d_t^j dj = 0$. By incorporating equations (13), (14), (15), and the assumption of i.i.d. productivity shocks, this condition can be rewritten as (see the online Appendix)

$$\underbrace{F(\bar{a}_t) \cdot \beta e_t}_{\text{agg. credit supply}} = \underbrace{\frac{\theta}{1-\theta} \cdot \int_{a>\bar{a}_t} dF(a) \cdot \beta e_t}_{\text{agg. credit demand}}, \tag{18}$$

where the left-hand side is the aggregate supply of credit (from entrepreneurs with $a_t^j < \bar{a}_t$) and the right-hand side is the aggregate demand of credit (from entrepreneurs with $a_t^j > \bar{a}_t$). By canceling the βe_t term on both sides, we get a simple equation that determines $\bar{a}_t = \bar{a}_n$, which is the time-invariant solution to the following equation:

$$F(\bar{a}_n) = \frac{\theta}{1-\theta}\left(1 - F(\bar{a}_n)\right). \tag{19}$$

Given that F is the CDF of a Pareto distribution over $[1, \infty)$ with shape parameter σ, this equation gives a closed-form solution for $\bar{a}_n$:

$$\bar{a}_n = \left(\frac{1}{1-\theta}\right)^{1/\sigma}. \tag{20}$$

The cutoff threshold is a proxy for allocation efficiency. Intuitively, a greater $\bar{a}_n$ is associated with less financial friction (a greater θ), implying more resources can be allocated to a more productive set of entrepreneurs. When $\theta \rightarrow 0$, the credit market shuts down, and $\bar{a}_n \rightarrow 1$, which is the lower bound of the distribution, implying that even the least productive entrepreneurs invest in capital. When $\theta \rightarrow 1$, there is no financial friction, and $\bar{a}_n \to \infty$, as only the most productive entrepreneurs invest in capital.

Given the cutoff threshold, the evolution of the aggregate capital stock can be derived from (16), (17), and (19) as

$$(21) \qquad K_{t+1} = \int_0^1 k_{t+1}^j \, dj = \frac{\beta}{1-\theta} \int_{\bar{a}_n} a \, dF(a) \cdot \alpha K_t^{\alpha} L_t^{1-\alpha}.$$

The interest rate is then given by

$$R_{t,t+1} = \bar{a}_n R_{t+1}^k = \bar{a}_n \alpha K_{t+1}^{\alpha-1}.$$

Finally, the aggregate employment and equilibrium wage are determined by labor market conditions (8), (9), and (10).

C. *Bubbleless Steady State*

Given the equilibrium dynamics, the steady state with no bubbles can be derived as follows. Because of the assumption that the rigidity parameter is a constant $\gamma \leq 1$, the downward wage rigidity condition (8) does not bind in steady state, leading to full employment:

$$(22) \qquad L_n = 1.$$

Then, from (21) and (22), the aggregate capital stock can be expressed as a function of $\bar{a}_n$:

$$(23) \qquad K_n = (\mathcal{A}_n \alpha)^{\frac{1}{1-\alpha}},$$

where

$$(24) \qquad \mathcal{A}_n \equiv \frac{\beta}{1-\theta} \int_{\bar{a}_n} a \, dF(a).$$

From (13) and (23), the interest rate can also be expressed as a function of $\bar{a}_n$:

$$(25) \qquad R_n = \frac{\bar{a}_n}{\mathcal{A}_n},$$

In summary, equations (20), (22), (23), and (25) uniquely determine the bubbleless steady state.

III. Bubbly Equilibrium

We now analyze a stochastic bubbly equilibrium, where the financial asset is priced above its fundamental value of zero. The asset thus plays the role of a (pure) bubbly asset, as in Tirole (1985). Note that in practice, bubbles are typically attached to stock and housing assets; it is, however, difficult to model such bubbles in the infinite-horizon framework (see Miao and Wang 2018 and the related estimated model in Miao, Wang, and Xu 2015a).

To model a stochastic bubble, we follow the literature (e.g., Weil 1987) and focus on equilibria where in each period the bubble persists ($p_t^b > 0$) with a probability $\rho \in (0,1)$ and permanently collapses ($p_{t+j}^b = 0, \forall j \geq 0$)[6] with the complementary probability $1 - \rho$, where a lower ρ means a riskier bubble. The two sources of uncertainty in the model are thus the idiosyncratic productivity shock and the aggregate bubble shock that makes the bubble collapse. We focus on the relevant parameter range in which the DWR is slack as long as the bubble persists. Detailed derivations are relegated to the online Appendix.

A. *While the Bubble Persists*

Optimal Decisions of Individual Entrepreneurs.—Suppose the bubble persists in t, i.e., $p_t^b > 0$. Then, while the optimal consumption of each entrepreneur is still a fraction $1 - \beta$ of net worth as in equation (11), her portfolio optimization will include a new decision of speculating on the bubbly asset:

$$I_t^j + \left(-d_t^j\right) + p_t^b b_t^j = \beta e_t^j.$$

On the one hand, the savings options of investing in capital and lending yield riskless returns of $a_t^j R_{t+1}^k$ and $R_{t,t+1}$, respectively. As in the bubbleless benchmark, the bubbly equilibrium will feature a cutoff productivity threshold $\bar{a}_t$ in each period such that: all entrepreneurs with productivity shocks below this threshold will not invest in capital (i.e., the constraint $I_t^j \geq 0$ binds), and all those with productivity shocks above it will only invest in capital, sell all of their assets, and borrow as much as possible (i.e., the credit constraint binds). Thus, the entrepreneurial capital investment decision and the amount of capital produced are given by equations (15) and (16), respectively, as in the bubbleless benchmark.

On the other hand, the speculative investment in the bubbly asset yields a risky return that is zero with probability $1 - \rho$. In the bubbly equilibrium, the less productive entrepreneurs must be willing to both lend and purchase the bubbly assets, and so they must be indifferent between the two options. Thus, the net debt position of entrepreneurs is given by

$$(26) \qquad d_t^j = \begin{cases} -\beta e_t^j + p_t^b b_t^j & \text{if } a_t^j < \bar{a}_t \\ \beta \frac{\theta}{1-\theta} e_t^j & \text{if } a_t^j > \bar{a}_t \end{cases},$$

the bubbly investment is given by

$$(27) \qquad p_t^b b_t^j = \begin{cases} \beta e_t^j + d_t^j & \text{if } a_t^j < \bar{a}_t \\ 0 & \text{if } a_t^j > \bar{a}_t \end{cases},$$

[6] That is, once collapsed, bubbles are not expected to reemerge. As in Guerron-Quintana, Hirano, and Jinnai (2018), the model can be extended to relax this assumption and allow for recurring bubbles.

and the indifference condition, which determines the growth rate of the price of the bubbly asset, is given by

$$E_t\left[u'\left(c^j_{t+1}\right)\left(\frac{p^b_{t+1}}{p^b_t} - R_{t,t+1}\right)\right] = 0, \quad \text{if } a^j_t < \bar{a}_t. \tag{28}$$

In addition, the marginal investors are indifferent between lending and investing in capital:

$$E_t\left[u'\left(c^j_{t+1}\right)\left(a^j_t R^k_{t+1} - R_{t,t+1}\right)\right] = 0, \quad \text{if } a^j_t = \bar{a}_t. \tag{29}$$

The net worth of each entrepreneur now also contains the value of the bubbly assets purchased from last period:

$$e^j_t = R^k_t a^j_{t-1} I^j_{t-1} - R_{t-1,t} d^j_{t-1} + \underbrace{p^b_t b^j_{t-1}}_{\text{bubbly component}}.$$

Intuitively, the bubbly asset provides an additional investment vehicle for entrepreneurs. When they are less productive, they can invest in the bubbly asset. Then when they become more productive, they sell the asset in order to make more capital investment.

Remark 1: The optimal decisions of each entrepreneur necessarily satisfy the transversality condition $\lim_{t\to\infty} E_0 \beta^t u'\left(c^j_t\right) p^b_t b^j_t = 0$. In a representative agent model, this condition can be used to rule out the possibility of bubbles (e.g., Kamihigashi 2001). Intuitively, the condition imposes that the present discounted value of the individual investment in the bubbly asset $p^b_t b^j_t$ must be zero. In a representative model, because $b^j_t = b_t = 1$, this condition implies that the present discounted value of the total value of the bubbly asset p^b_t must be zero. However, with heterogeneous entrepreneurs and occasionally binding credit constraints, the individual bubbly investment is *not* the same as the total value of the bubbly asset ($p^b_t b^j_t \neq p^b_t$), because entrepreneurs have heterogeneous bubbly investment positions (recall equation (27)). Therefore, in a heterogeneous agent model with incomplete markets like ours (or Kocherlakota 2009 and Hirano and Yanagawa 2017), the individual transversality condition does not rule out the possibility of bubbles in equilibrium (see Kocherlakota 1992 for a more general exposition of this point).

Aggregation.—Aggregate variables of the bubbly economy evolve as follows. The aggregate net worth of entrepreneurs now consists of not only capital income but also the value of the bubbly asset:

$$e_t = \int_0^1 e^j_t \, dj = \alpha K^\alpha_t L^{1-\alpha}_t + p^b_t. \tag{30}$$

The right-hand side of this equation highlights the bubble's crowd-in effect: the bubble resale value p^b_t helps increase the net worth of entrepreneurs in equilibrium.

The cutoff threshold $\bar{a}_t$ is determined by the credit market clearing condition $\int_0^1 d_t^j dj = 0$, or equivalently (see the online Appendix):

$$\underbrace{F(\bar{a}_t) \cdot \beta e_t - p_t^b}_{\text{agg. credit supply}} = \underbrace{\frac{\theta}{1-\theta} \cdot \left(1 - F(\bar{a}_t)\right) \cdot \beta e_t}_{\text{agg. credit demand}}.$$

The left-hand side of this equation highlights the bubble's crowd-out effect: the aggregate speculative investment in the bubbly asset (p_t^b) crowds out the flow from aggregate savings $(F(\bar{a}_t)\beta e_t)$ into the supply side of the credit market. By canceling e_t on both sides and defining the *bubble* (*over savings*) *ratio* as

$$\phi_t \equiv \frac{p_t^b}{\beta e_t},$$

the equation above can be rewritten as

$$F(\bar{a}_t) - \phi_t = \frac{\theta}{1-\theta} \cdot \left(1 - F(\bar{a}_t)\right). \tag{31}$$

Note that ϕ_t necessarily lies in $(0,1)$.

From (30) and (31), the aggregate capital stock evolves according to

$$K_{t+1} = \int_0^1 k_{t+1}^j dj = \frac{\beta}{1-\theta} \cdot \int_{\bar{a}_t} a\, dF(a) \cdot \left(\alpha K_t^\alpha L_t^{1-\alpha} + p_t^b\right). \tag{32}$$

Furthermore, indifference conditions (28) and (29) determine the interest rate and the growth of the bubbly asset, which are derived in the online Appendix as

$$R_{t,t+1} = \bar{a}_t R_{t+1}^k = \bar{a}_t \alpha K_{t+1}^{\alpha-1} \tag{33}$$

and

$$\frac{\phi_{t+1}}{\phi_t} = \frac{\left(1 - \beta\phi_{t+1}\right)\bar{a}_t}{\frac{\beta}{1-\theta}\int_{\bar{a}_t} a\, dF(a)} \frac{F(\bar{a}_t) - \phi_t}{\rho F(\bar{a}_t) - \phi_t}. \tag{34}$$

Finally, the aggregate employment and equilibrium wage are determined by labor market conditions (8), (9), and (10).

Stochastic Bubbly Steady State.—We now characterize the stochastic bubbly steady state. Credit-clearing condition (31) implies the bubble ratio ϕ as a function of $\bar{a}_b$:

$$F(\bar{a}_b) - \phi = \frac{\theta}{1-\theta} \cdot \left(1 - F(\bar{a}_b)\right). \tag{35}$$

The only difference between equation (35) and its counterpart (19) in the bubbleless benchmark is the presence of ϕ on the left-hand side, representing the fact that in the bubbly economy, relatively less productive entrepreneurs have the bubbly asset as an additional investment vehicle besides lending in the credit market. From (35) we get a closed-form expression for $\bar{a}_b$:

$$\bar{a}_b = \left(\frac{1}{(1-\theta)(1-\phi)}\right)^{1/\sigma} > \bar{a}_n = \left(\frac{1}{1-\theta}\right)^{1/\sigma}. \tag{36}$$

The fact that $\phi > 0$ implies $\bar{a}_b > \bar{a}_n$. Thus, even though at the individual level entrepreneurs may not see an advantage of having a bubble, at the aggregate level the buying and selling of the bubbly asset allows for more resources to be transferred from less productive to more productive entrepreneurs. The bubble causes the productivity threshold to rise from $\bar{a}_n$ to $\bar{a}_b$, reflecting a more efficient allocation. As a consequence, the average entrepreneurial productivity is higher during a bubbly episode, which is consistent with empirical observations (Miao and Wang 2012).

As in the bubbleless steady state, given the assumption that the rigidity parameter is a constant $\gamma \leq 1$, the downward wage rigidity condition (8) does not bind in steady state, leading to

$$L_b = 1. \tag{37}$$

Then, from (32) and (37), the aggregate capital stock can be expressed as a function of $\bar{a}_b$ and ϕ:

$$K_b = \left(\frac{\frac{\beta}{1-\theta}\alpha}{1-\beta\phi}\int_{\bar{a}_b} a\,dF(a)\right)^{\frac{1}{1-\alpha}}. \tag{38}$$

From (29) and (38), the interest rate can also be expressed as a function of $\bar{a}_b$ and ϕ:

$$R_b = \frac{(1-\beta\phi)\bar{a}_b}{\frac{\beta}{1-\theta}\int_{\bar{a}_b} a\,dF(a)}. \tag{39}$$

Finally, from indifference condition (34) and from (39), the steady-state bubble ratio also has a closed-form solution:

$$\begin{aligned} \phi &= \frac{\rho - R_b}{1-R_b}F(\bar{a}_b) \\ &= \frac{\theta}{\beta}\frac{1-(1-\beta\rho)\sigma}{\theta+\sigma(1-\theta)(1-\rho)}. \end{aligned} \tag{40}$$

Equations (36) to (40) characterize the endogenous variables in the stochastic bubbly steady state. Given (40), the condition for the existence of a bubbly steady state can be characterized as follows.

LEMMA 2: *A bubbly steady state exists if and only if*

$$\frac{\sigma-1}{\beta\sigma} < \rho. \tag{41}$$

PROOF:

For the variables characterized by equations (36) to (40) to constitute a bubbly steady state, a necessary and sufficient condition is the bubble ratio satisfies $\phi \in (0,1)$. From (40), $\phi > 0$ equivalent to $1 - (1 - \beta\rho)\sigma > 0$, i.e., (41). And once this condition is satisfied, it is immediately true that $\phi < 1$. ■

The condition implies that for a stochastic bubble to exist, the probability that the bubble persists ρ has to be sufficiently high (as otherwise agents in the economy would deem the bubble to be too risky as an investment vehicle). Another direct corollary of (40) and (41) is that ϕ is strictly increasing in θ, implying that a more relaxed credit constraint is associated with a larger bubble size in equilibrium.

For the rest of the paper, we will impose the bubble existence condition (41). Furthermore, as in the recent literature, we will focus on the relevant range of parameters in which the bubble is expansionary (the crowd-in effect dominates the crowd-out effect in steady state), that is,

$$K_b > K_n, \tag{42}$$

where the stochastic bubbly steady-state capital stock K_b is given by (38) and the bubbleless steady-state capital K_n is given by (23).[7]

B. *Post-bubble Dynamics*

We now study the effect of the collapse of the bubble on the economy, which is the main focus of the paper. Suppose the bubble collapses at a certain period T, i.e., $p^b_{T+s} = 0, \forall s \geq 0$. As the expansionary effect of the bubble ends, the post-bubble capital stock and wage will decline toward the bubbleless steady-state levels. However, if the downward wage rigidity constraint binds, then the wage cannot flexibly fall to clear the labor market. Instead, employment is determined by the demand of firms. The rigidly high wage thus leads to involuntary unemployment. The contraction in employment reduces the return from capital and entrepreneurs' net worth. Both of these effects in turn reduce entrepreneurs' accumulation of capital. The wage rigidity thus amplifies and propagates the shock of bursting bubbles.

Let $T + s^*$ be the first post-bubble period when full employment is recovered, i.e.,

$$s^* \equiv \min\{s \in \mathbb{N} | L_{T+s} = 1\},$$

where $\mathbb{N} \equiv \{0, 1, 2, \ldots\}$. If $s^* > 0$, then we say the economy is in a slump between $T + 1$ and $T + s^* - 1$, as there is *involuntary unemployment* during this period. Given the tractability of the model, we can analytically characterize the duration of the slump. Intuitively, the economy escapes the slump when the equilibrium wage has fallen enough that the downward wage rigidity no longer binds.

[7] Written in exogenous parameters, this assumption is equivalent to $(1 - \phi)^{\frac{\sigma - 1}{\sigma}} > 1 - \beta\phi$, where ϕ is given by (40).

PROPOSITION 3 (Post-bubble Slump): *Suppose the bubble collapses in period T. Then the economy enters a slump for s^* periods. The post-bubble equilibrium dynamics are given by* (7) *and*

$$K_{T+s+1} = \left[\mathcal{A}_n \alpha K_T^{\alpha-1}\right]^{s+1} \gamma^{\frac{\alpha-1}{\alpha}\frac{s(s+1)}{2}} K_T, \tag{43}$$

$$w_{T+s} = \gamma^s w_T, \quad \forall\, 0 \leq s < s^*, \tag{44}$$

and the duration of the slump is given by

$$s^* = \max\left\{0, \left\lceil 2\alpha \log_{\frac{1}{\gamma}}\left(\frac{K_T}{K_n}\right) - \frac{1+\alpha}{1-\alpha}\right\rceil\right\}, \tag{45}$$

where the ceiling function $\lceil x \rceil$ *denotes the least integer greater than or equal to x. The economy regains full employment and follows the dynamics of Section II for* $t \geq T + s^*$.

PROOF:

By the labor market clearing conditions during the slump, the wage rigidity must bind in all periods for which $L_{T+s} < 1$. Thus, during the slump, the equilibrium wage is given by (44). From (7), (21), and (44), the post-bubble capital stock's law of motion can be written as

$$K_{T+s+1} = \mathcal{A}_n \alpha \left(\frac{\gamma^s w_T}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} K_{T+s},$$

whose recursion leads to

$$K_{T+s+1} = \left[\mathcal{A}_n \alpha \left(\frac{w_T}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}}\right]^{s+1} \gamma^{\frac{\alpha-1}{\alpha}\frac{s(s+1)}{2}} K_T.$$

By substituting in (7) and $L_T = 1$ for w_T, we then get (43), as desired.

To determine the duration s^* of the slump, recall

$$\begin{aligned} s^* &\equiv \min\left\{s \in \mathbb{N} \mid L_{T+s} = 1\right\} \\ &= \min\left\{s \in \mathbb{N} \mid w_{T+s}^f \geq \gamma w_{T+s-1}\right\}, \end{aligned}$$

where $w_{T+s}^f = (1-\alpha)K_{T+s}^{\alpha}$ represents the wage level consistent with full employment. Then we can rewrite s^* as

$$s^* = \min\left\{s \in \mathbb{N} \mid (1-\alpha)K_{T+s}^{\alpha} \geq \gamma^s w_T\right\}.$$

Algebraic manipulation yields (45). ∎

Numerical Illustration.—We conduct a simple calibrated numerical exercise to illustrate the equilibrium dynamics. Since the model is intentionally designed to be stylized and parsimonious, this exercise should not be viewed as a full-fledged quantitative analysis but rather a suggestive quantitative illustration of the model's predictions. In this section, we also make two basic extensions to improve the mapping of the model to data. First, we assume the economy grows at an exogenous rate $g \geq 0$. Second, we assume capital partially depreciates at rate $\delta \in [0,1]$ (see the online Appendix for details).

We then calibrate the model to Japanese data as follows. We will choose parameters to match the pre-bubble phase (1970–1986) and the boom phase (the bubble period of 1987–1991) and let the model predict the bust phase (post-1991). There are two sets of parameters, the first of which can be set using relatively standard values from the literature. Specifically, we set a period to be a year, the capital share to be $\alpha = 0.33$, the discount factor to be $\beta = 0.96$, the capital depreciation rate to be $\delta = 0.076$, and the exogenous growth rate to be $g = 0.04$. Following Schmitt-Grohé and Uribe (2016), the downward wage rigidity parameter is set to be close to one: $\gamma = 0.961$. The second set of parameters, consisting of shape parameter σ of the productivity distribution, the financial friction parameter θ, and the bubble persistence parameter ρ, are less standard and will be calibrated. In particular, we choose σ, θ, and ρ to match R_n, K_n/Y_n, and $(K_b + p_b)/Y_b$ to three moments: the average real interest rate of 1.02 in Japan in the pre-bubble phase, the wealth-over-income ratios in the pre-bubble phase and in the bubble phase of 3.67 and 5.18, respectively.[8] The calibrated parameter values are $\sigma = 17.089$, $\theta = 0.096$, and $\rho = 0.999$.

Figure 1 illustrates a simulated equilibrium path for detrended aggregate variables under this parametrization. On this path, we set the economy at the stochastic bubbly steady state in the initial period, and then the bubble collapses in $t = 10$ (in the simulation, agents rationally expect that the bubble is stochastic and can burst in any period). Equilibrium variables are plotted with the solid lines, and for comparison, the bubbleless steady-state counterparts are plotted with dashed lines. As seen in the figure, as long as the bubble lasts, the economy experiences a boom in entrepreneurial net worth (relative to the bubbleless steady state), which leads to a boom in aggregate credit to entrepreneurs and consequently a boom in aggregated capital accumulation, output, wage, and consumption.[9] Since the boom in the capital stock and bubble value is larger than that in output, the wealth over output ratio also increases during the bubbly episode.

However, after the bubble collapses, the economy begins a contraction. Without nominal rigidities, the labor market would be flexible and the equilibrium wage would simply decline back to the bubbleless steady-state level. However, with downward wage rigidity, the post-bubble equilibrium wage may not flexibly fall to clear the labor market, leading to involuntary unemployment. The drop in employment not only reduces the economy's output but also has important intertemporal

[8] Data for the wealth-over-income ratios come from Piketty and Zucman (2014); data for GDP and real interest rate come from the World Bank; the dating of the bubble period is according to Shioji (2013).

[9] Note that the boom in consumption is more pronounced for entrepreneurs, implying that entrepreneurs tend to gain more from the bubble than workers (as the increase in net worth allows entrepreneurs to increase their investment). This asymmetry could lead to interesting political economy implications, which are absent from this model and are left for future research.

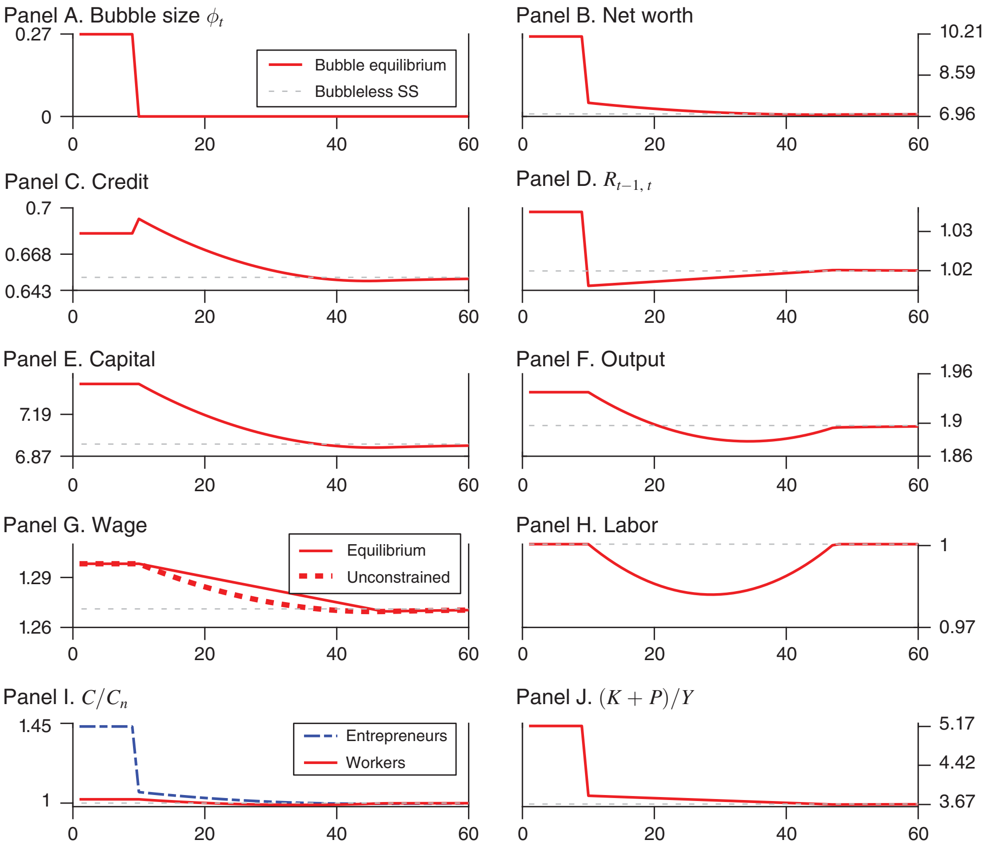

Figure 1. Equilibrium Dynamics with Bubble Boom-bust

Note: Solid lines represent detrended equilibrium variable values; gray dashed horizontal lines represent the corresponding bubbleless steady-state values.

effects. First, it reduces the net worth of entrepreneurs. Second, it reduces the return rate on capital. Both of these effects depress capital accumulation, explaining the contractions of aggregate economic activities during the slump with involuntary unemployment.

As a consequence, aggregate output, net worth, capital, credit, and consumption can *undershoot* (i.e., drop below) the pre-bubble trend. The figure highlights the boom-bust trade-off: the bubble leads to a boom of about 2.1 percent in output (relative to the bubbleless steady state) as long as it persists, but its collapse leads to a recession, where the aggregate output drops as much as 1.0 percent below the bubbleless steady state. The economy experiences long "lost decades": about 20 years of declining output, which only recovers to its bubbleless trend after about 40 years.

IV. Welfare and Policy Analysis

We will now investigate the welfare effects of stochastic bubbles. We define welfare as the steady-state lifetime expected utility.

A. *Workers*

We start with the welfare of workers in the bubbleless steady state, which is simply

$$W_n = \frac{\log(w_n)}{1-\beta} = \frac{\log\left[(1-\alpha)\,K_n^{\alpha}\right]}{1-\beta}. \tag{46}$$

The welfare of workers in the stochastic bubbly steady state features a boom-bust trade-off. As long as the bubble persists, their consumption is larger than that in the bubbleless steady state: $c_b^w = (1-\alpha)\,K_b^{\alpha} > c_n^w = (1-\alpha)K_n^{\alpha}$. However, after the bubble collapses, the economy enters a slump for s^* periods, during which workers suffer from involuntary unemployment. In the online Appendix, we show that the welfare of workers in the stochastic bubbly steady state is given by

$$W_b = \underbrace{\frac{\log\left[(1-\alpha)K_b^{\alpha}\right]}{1-\rho\beta}}_{\text{expected utility when bubble persists}} + \underbrace{\frac{\beta(1-\rho)}{1-\rho\beta}\left[\Gamma_0(s^*) + \Gamma_1(s^*)\log K_b\right]}_{\text{expected utility after bubble collapses}}, \tag{47}$$

where the expressions for Γ_0 and Γ_1 are provided in the online Appendix.

It is clear that W_b depends on the bubble's risk of bursting $1-\rho$ and on the duration of the post-bubble slump s^*, which is itself a function of the degree of wage rigidity γ (recall (45)). Since the slump length s^* is increasing in the degree of rigidity γ, W_b is decreasing in γ. Similarly, W_b is increasing in the persistence probability ρ, i.e., a safer bubble yields a higher payoff.

B. *Entrepreneurs*

The welfare of entrepreneurs is more complex, due to their heterogeneity and portfolio optimization. In the bubbleless steady state, the lifetime expected utility of an entrepreneur j who starts the period with a net worth e^j, denoted by $V_n(e^j)$, satisfies the following equation:

$$\begin{aligned} V_n(e^j) = {} & \underbrace{\log\left((1-\beta)\,e^j\right)}_{\text{current period utility}} \\ & + \beta \underbrace{F(\bar{a}_n)V_n\left(R_n\beta e^j\right)}_{\text{continuation value if } a^j \le \bar{a}_n} \\ & + \beta \underbrace{\int_{\bar{a}_n} V_n\left(\frac{aR_n^k - \theta R_n}{1-\theta}\,\beta e^j\right) dF(a)}_{\text{continuation value if } a^j > \bar{a}_n}. \end{aligned} \tag{48}$$

The online Appendix provides an analytical solution to this equation. To streamline the analysis, let us assume that each entrepreneur starts the bubbleless steady state with an equal net worth, leading to $e^j = \alpha K_n^\alpha$. Then the bubbleless steady-state entrepreneurial welfare is simply given by

$$V_n \equiv V_n(\alpha K_n^\alpha).$$

Similarly, in the bubbly steady state, lifetime expected utility of an entrepreneur j that starts the period with a net worth e^j, denoted by $V_b(e^j)$, satisfies the following equation:

$$(49) \quad V_b(e^j) = \underbrace{\log((1-\beta)\, e^j)}_{\text{current period utility}}$$
$$+ \rho\beta \underbrace{\left\{ F(\bar{a}_b) V_b(\rho\beta e^j) + \int_{\bar{a}_b} V_b\left(\frac{aR_b^k - \theta R_b}{1-\theta}\,\beta e^j\right) dF(a) \right\}}_{\text{if bubble persists}}$$
$$+ (1-\rho)\beta \underbrace{\left\{ F(\bar{a}_b) V_{burst}\left(\frac{F(\bar{a}_b) - \phi}{F(\bar{a}_b)} R_b \beta e^j\right) + \int_{\bar{a}_b} V_{burst}\left(\frac{aR_b^k - \theta R_b}{1-\theta}\beta e^j\right) dF(a) \right\}}_{\text{if bubble bursts}},$$

where $V_{burst}(\cdot)$ denotes the continuation value after the bubble bursts. The online Appendix provides analytical solutions to $V_b(\cdot)$ and $V_{burst}(\cdot)$.

As in the bubbleless case, we assume for simplicity that each entrepreneur starts the bubbly steady state with an equal net worth, leading to $e^j = \alpha K_b^\alpha/(1-\beta\phi)$. Then the bubbly steady-state entrepreneurial welfare is given by

$$V_b \equiv V_b\left(\frac{\alpha K_b^{1-\alpha}}{1-\beta\phi}\right).$$

From Sections IVA and IVB, we can show that if the bubble is sufficiently risky, and if there is sufficient wage rigidity, then agents in the economy are better off if there were no stochastically bursting bubbles. This is intuitive as the welfare gain from a short and small boom (because ρ is small) is dominated by the loss from a long and severe post-bubble slump (because γ is large).

PROPOSITION 4 (Welfare-Reducing Stochastic Bubble): *Hold ρ fixed and assume $\rho < \bar{\rho} \equiv 1 - (\alpha(1-\beta)^2/\beta(\beta-\alpha))$ (the bubble is sufficiently risky). There exists $\bar{\gamma} \in (0,1)$ such that if $\gamma > \bar{\gamma}$ (wage is sufficiently rigid), then the bubble reduces steady-state welfare:*[10]

$$W_b < W_n, \quad V_b < V_n.$$

[10] As a simple numerical illustration, assuming $\alpha = 0.33$, $\beta = 0.96$, then the proposition implies that if $\gamma = 1$ (wages cannot decline), agents in the economy will be better off without any stochastic bubble with the probability of persisting ρ smaller than $\bar{\rho} = 0.999$.

PROOF:

Appendix A.A1. ∎

C. *Leaning-against-the-bubble Policy*

The fundamental source of inefficiencies in the model is a form of "bubbly pecuniary externality": individual entrepreneurs do not internalize the general equilibrium effects of their portfolio choices in driving a boom in asset prices. Under this context, policy responses are warranted. We analyze a macroprudential policy of taxing bubble speculation, so that private agents internalize the pecuniary externality. As we will show, this policy has an effect of reducing the bubble size, and is thus akin to the kind of "leaning-against-the-wind" policies that have been extensively discussed in the policy circle (e.g., Barlevy 2012, 2018), and is similar to the type of tax policies often considered in the macroprudential literature (e.g., Lorenzoni 2008; Gertler, Kiyotaki, and Queralto 2012; Jeanne and Korinek 2013).[11]

Formally, consider a benevolent constrained policymaker who cares about the welfare of both workers and entrepreneurs. Throughout, we assume that the policymaker takes the bubble risk ρ as given, i.e., it cannot select a less risky bubbly equilibrium. Furthermore, the constrained policymaker cannot undo the friction in the credit market (e.g., via redistribution) nor undo friction in the labor market. However, it can levy a tax τ on the return from bubble speculation. As our focus is on steady-state welfare, we assume that the tax rate is constant. Then, the budget constraint (1) becomes

$$c_t^j + I_t^j + p_t^b b_t^j = R_t^k k_t^j + d_t^j - R_{t-1,t} d_{t-1}^j + (1-\tau) p_t^b b_{t-1}^j + T_t^j. \tag{50}$$

The after-tax return on bubble speculation for the entrepreneur is then $(1-\tau)p_{t+1}^b/p_t^b$. The policymaker rebates the tax revenue back to the entrepreneur through a lump-sum transfer:

$$T_t^j = \tau p_t^b b_{t-1}^j, \tag{51}$$

which the entrepreneur takes as given.

Remark 5: Consistent with the aforementioned notion of constrained policymaking, this specification of tax and transfer implies that the policymaker cannot redistribute resources across entrepreneurs or between entrepreneurs and workers. The reason we rule out redistribution policies is as follows. There are two sources of inefficiencies in our model: (i) the pecuniary externality in the speculative bubbly investment, and (ii) the misallocation due to heterogeneous productivities and financial friction. The way we model a constrained macroprudential policymaker, who

[11] As in most of the literature, we implicitly assume that policymakers can observe the bubble. Of course, this is a strong assumption. Alternatively, one can interpret the macroprudential policy as imposing a tax on speculative investments in broad classes of assets that are ex ante perceived to be likely to experience bubbles, such as real estate or stocks of certain types of companies.

can impose a macroprudential tax on speculative investment but cannot redistribute, ensures that the sole objective of the macroprudential policy is to correct for the pecuniary externality, which is the focus of the paper. This assumption allows us to zoom in on the effect of the macroprudential policy on the pecuniary externality, in the same spirit as the pecuniary externality literature (e.g., Farhi and Werning 2016).

The online Appendix derives the bubbly equilibrium dynamics and steady state with the tax. A key result is that the steady-state bubble size is a decreasing function of the macroprudential tax τ:

$$\phi(\tau) = \frac{\theta}{\beta} \cdot \frac{1-\sigma\big(1-\beta\rho(1-\tau)\big)}{\theta-\sigma\rho(1-\theta)(1-\tau)+\sigma(1-\tau-\theta)} \leq \phi. \tag{52}$$

When $\tau = 0$, the bubble size collapses to $\phi(0) = \phi$, which is the laissez-faire size as derived in Section III. Hence, the tax not only makes the bubble smaller, but it also makes the bubble harder to arise. Specifically, under the tax, a bubbly steady state exists if and only if

$$\frac{\sigma-1}{\beta\sigma} < \rho(1-\tau),$$

which is more stringent than the laissez-faire existence condition (41). Thus, by setting $\tau \geq \bar{\tau} \equiv 1-((\sigma-1)/\beta\sigma\rho)$, the policymaker can effectively rule out the possibility of a bubbly equilibrium.[12]

Figure 2 illustrates an equilibrium path with and without the tax. The dashed lines represent the laissez-faire equilibrium path (as plotted in Figure 1), while the solid lines represent the economy under a macroprudential tax of $\tau = 1$ percent. As shown in the figure, the tax effectively reduces the bubble size. There is a boom-bust trade-off: the policy mitigates the effects of a collapsing bubble (the slump is shorter and less severe), but it also reduces the boom in aggregate economic activities while the bubble lasts.

To evaluate the policy, the online Appendix also derives the bubbly steady-state welfare expressions for both workers and entrepreneurs under the tax, denoted by $W_b(\tau)$ and $V_b(\tau)$, respectively. Assume the policymaker assigns a Pareto weight $\lambda \in [0,1]$ on the welfare of workers (and $1-\lambda$ on the welfare of entrepreneurs). Given a fixed ρ, a *constrained optimal policy* is a macroprudential tax τ that maximizes the Pareto-weighted bubbly steady-state welfare:[13]

$$\max_{\tau \leq \bar{\tau}} \lambda W_b(\tau) + (1-\lambda) V_b(\tau).$$

Due to the highly nonlinear behaviors of W_b and V_b, in general, the optimal tax can only be solved for with numerical methods. However, an interesting implication

[12] For the parameter values used in Section IIIB, the associated value for $\bar{\tau}$ is 1.8 percent.

[13] It is straightforward to show that a constrained-optimal policy implements a constrained-efficient allocation, where the notion of constrained efficiency is defined in Section A.5 of the online Appendix, following the macroprudential literature.

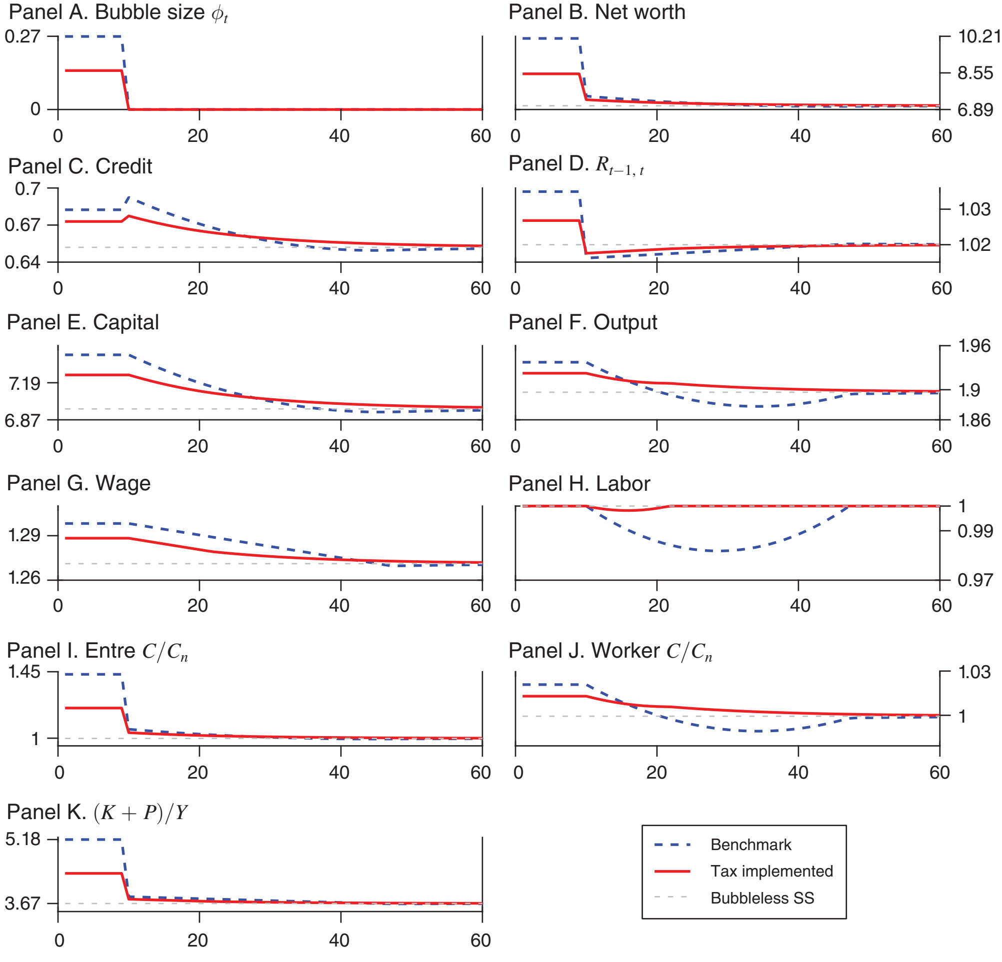

FIGURE 2. EQUILIBRIUM DYNAMICS WITH BUBBLE BOOM-BUST

Note: Solid and dashed lines represent detrended equilibrium variable values with and without tax, respectively; gray dashed horizontal lines represent the corresponding bubbleless steady-state values.

of our previous analysis is that when the conditions of Proposition 4 are met, an optimal policy is to rule out the possibility of the bubble altogether by setting $\tau = \bar{\tau}$. Formally, we have the following result.

COROLLARY 6: *There exists $\bar{\gamma} \in (0,1)$ such that if there is sufficient wage rigidity $(\gamma \geq \bar{\gamma})$ and the bubble is sufficiently risky $\left(\rho < \bar{\rho} \equiv 1-\left(\alpha(1-\beta)^2/\beta(\beta-\alpha)\right)\right)$, then a constrained-optimal macroprudential tax is to set*

$$\tau = \bar{\tau},$$

which effectively rules out the possibility of a bubble.

PROOF:

Appendix A.A2. ∎

V. Zero Lower Bound

We now extend the real model by introducing downward *nominal* wage rigidity (DWNR) and a nominal interest rule that is constrained by the zero lower bound (ZLB). We will show that the collapse of a large bubble can push the interest rate against the ZLB and the push economy into a "secular stagnation."

DNWR: Formally, let P_t denote the price level of the consumption good in period t in unit of a currency, and let w_t continue to denote the real wage. Instead of the real wage rigidity condition (8), we impose the following assumption on nominal wages (à-la Schmitt-Grohé and Uribe 2017):

$$P_t w_t \geq \gamma(L_t) P_{t-1} w_{t-1}, \quad \forall t \geq 1,$$

where the degree of rigidity γ is now a function of L_t:

$$\gamma(L) \equiv \gamma_0 L^{\gamma_1}, \quad \gamma_0, \gamma_1 > 0,$$

The fact that γ is increasing in L implies that nominal wages are more flexible as unemployment increases but more rigid as employment increases. Furthermore, as we will show, the assumption $\gamma_1 > 0$ implies that there could exist a "secular stagnation" bubbleless steady state that features involuntary unemployment. The nominal wage rigidity condition can be rewritten as

$$w_t \geq \frac{\gamma(L_t)}{\Pi_{t-1,t}} w_{t-1}, \tag{53}$$

where $\Pi_{t-1,t} \equiv P_t / P_{t-1}$ is the gross inflation rate between $t-1$ and t.

ZLB: As is standard in the literature, we assume that the entrepreneurs can trade nominal government bonds, which are in net zero supply and yield an interest rate $1 + i_{t,t+1}$. The rate is set according to a Taylor rule subject to a ZLB:

$$1 + i_{t,t+1} = \max\left\{1, R^f_{t,t+1} \left(\Pi_{t-1,t}\right)^{\zeta} \left(\Pi^*\right)^{1-\zeta}\right\}, \tag{54}$$

where $R^f_{t,t+1}$ is the real interest rate that would prevail with full employment in $t+1$ (i.e., $L_{t+1} = 1$), $\Pi^* > 0$ is an inflation target, and $\zeta > 1$ is a constant. The rule implies that if the ZLB does not bind, then the inflation would be stabilized at the target Π^*.[14]

[14] We do not model optimal monetary policy explicitly here. This is because, in our model, an increase in the inflation rate always weakly improves welfare by mitigating the wage rigidity. Thus, setting a very high inflation target to avoid involuntary unemployment, and the ZLB will be optimal. Realistically, there are costs of inflation, such as the costs associated with nominal price rigidities, that are not modeled explicitly here. Also, in practice, central banks tend to follow similar Taylor rules with inflation targets.

The definition of an equilibrium is similar to before, except that we have an additional endogenous variable P_t for the price level, the labor market complementary-slackness condition is

$$(1 - L_t)\left(w_t - \frac{\gamma(L_t)}{\Pi_{t-1,t}} w_{t-1}\right) = 0,$$

and the monetary policy rule (54) holds.

A. *Bubbleless Equilibrium and Multiple Steady States*

In the bubbleless equilibrium, the cutoff threshold and capital stock are again given by (19) and (21). The equilibrium wage, employment, and inflation satisfy

$$w_t = \max\left\{(1-\alpha)K_t^{\alpha}, \gamma(L_t)\frac{w_{t-1}}{\Pi_{t-1,t}}\right\}$$

and

$$\frac{\max\left\{1, R_{t,t+1}^{f}(\Pi_{t-1,t})^{\zeta}(\Pi^*)^{1-\zeta}\right\}}{\Pi_{t,t+1}} = R_{t,t+1}.$$

For the rest of the paper, we assume

$$\Pi^* > \gamma_0 > \frac{1}{R_n}, \tag{55}$$

where R_n is the bubbleless steady-state interest rate, as given by (25). Under this assumption, because of the kink in the Taylor rule and the fact that the degree of wage rigidity is a function of employment, there are two possible bubbleless steady states. In the "good" steady state (which will continue to be denoted with a subscript n), there is full employment ($L_n = 1$); the ZLB is slack; the inflation is at the target Π^*; the capital stock is given by K_n, as in (23); and the real interest rate is given by R_n, as in (25). There is another "bad" steady state, where the ZLB binds ($i = 0$) and inflation is below target, and there is involuntary unemployment ($\underline{L} < 1$), leading to a lower capital stock:

$$\underline{K} = K_n \underline{L} < K_n.$$

The real interest rate is given by the indifference condition of the marginal investor: $\underline{R} = \bar{a}_n \alpha \underline{K}^{\alpha-1}\underline{L}^{1-\alpha} = R_n$. The inflation rate is determined by the Fisher equation $\underline{R\Pi} = 1$, or equivalently,

$$\underline{\Pi} = \frac{\frac{\beta}{1-\theta}\int_{\bar{a}_n} a\, dF(a)}{\bar{a}_n},$$

which is smaller than the target Π^* under assumption (55). The employment level $\underline{L}$ is determined by the binding DNWR condition $1 = \gamma(L)/\Pi$, which gives

$$\underline{L} = (\underline{\Pi}/\gamma_0)^{\frac{1}{\gamma_1}}.$$

Assumption (55) guarantees that there is involuntary unemployment in this steady state ($\underline{L} < 1$), and the ZLB does indeed bind ($\underline{R}\,\Pi^{\zeta}(\Pi^*)^{1-\zeta} < 1$).

B. *Bubbly Equilibrium*

We now analyze the bubbly economy. We focus on the relevant parameter range in which the DNWR and the ZLB are slack as long as the bubble persists.[15] Then as inflation is stabilized at the target, the bubbly equilibrium dynamics are as characterized in Section III, and the steady state is as characterized in Section IIIA.

The post-bubble dynamics will, however, be different. Suppose the economy reaches the bubbly steady state and then the bubble collapses at period T (i.e., $p^b_{T+s} = 0, \forall s \geq 0$). The collapse of the bubble *exerts downward pressure on the real interest rate* through two channels. First, after the bubble collapses, the productivity of the marginal investor decreases from $\bar{a}_b$ to $\bar{a}_n$. Thus, instead of the identity $R_{T,T+1} = \bar{a}_b R^k_{T+1}$ that would have prevailed if the bubble did not collapse in T, the real interest is given by $R_{T,T+1} = \bar{a}_n R^k_{T+1}$, with $\bar{a}_n < \bar{a}_b$. Second, as the bubble has an expansionary effect on capital accumulation, the post-bubble economy will follow the bubbleless dynamics as specified in the previous section, but with an initial capital stock K_b, which is larger than that in the good steady state K_n. A high capital stock leads to a low marginal product of capital and thus a low interest rate. The combination of these two mechanisms exerts a downward pressure on the real interest rate and thus the nominal interest rate. If the bubble leads to sufficient large accumulation of capital stock, its collapse can push the interest rate against the ZLB. Formally, we have the following result.

PROPOSITION 7 (Effect of Bubble's Collapse on R): *Suppose the economy has reached the steady state with an expansionary bubble and then the bubble collapses in a period denoted by T. If the bubbly steady state K_b is sufficiently large such that*

$$K_b > \bar{K} \equiv \left(\bar{a}_n \mathcal{A}_n \Pi^*\right)^{\frac{1}{\alpha(1-\alpha)}} K_n,$$

then the Taylor rule (*54*) *is constrained by the ZLB*:

$$1 + i_{T,T+1} = 1 > R^f_{T,T+1}\left(\Pi_{T-1,T}\right)^{\zeta}\left(\Pi^*\right)^{1-\zeta}. \tag{56}$$

PROOF:

Appendix A.A3. ∎

Remark: One could think of this as corresponding to a situation of "investment hangover," or capital overinvestment, at the end of an economic boom (Simsek, Shleifer, and Rognlie 2014). The difference between our paper and Simsek, Shleifer, and Rognlie (2014) is that the overinvestment is endogenous in our framework, while it is imposed exogenously in theirs.

[15] This is the case when the initial capital stock K_0 and the initial bubble value p^b_0 are below the bubbly steady-state levels, and $R_b > 1/\Pi^*$, where R_b is given by (39).

The next result shows that in the post-bubble economy, whenever the ZLB binds, the DNWR must also bind.

LEMMA 8 (ZLB implies DNWR): *For any* $t \geq T+1$, *if* $i_{t-1,t} = 0$ *then* $L_t < 1$.

PROOF:
Appendix A.A4. ∎

We say that the economy is in a *liquidity trap* in period t if the ZLB binds (implying $i_{t-1,t} = 0$) and the DNWR binds (implying $L_t < 1$). We now show a stark result that under certain conditions, the post-bubble economy may *never* escape from the liquidity trap.[16] Specifically, we will construct a post-bubble equilibrium path where $L_t < 1$ and $i_{t-1,t} = 0$ for all $t \geq T+1$.

PROPOSITION 9 (Post-bubble Secular Stagnation): *Let* $\{K_{T+t}, L_{T+t}, \Pi_{T+t-1,T+t}\}$ *be defined by the following closed-form expressions:*

$$K_{T+t} = \left(\mathcal{A}_n \alpha\right)^{\frac{1-\alpha^t}{1-\alpha}} \left(K_T\right)^{\alpha^t} \left(\frac{\mathcal{A}_n}{\gamma_0 \bar{a}_n}\right)^{\frac{1-\alpha}{\gamma_1}\left(\frac{1-\alpha^{t-1}}{1-\alpha} - \frac{1-\left((1+\gamma_1)\alpha\right)^{t-1}}{\left(1-(1+\gamma_1)\alpha\right)(1+\gamma_1)^{t-1}}\right)},$$

$$L_{T+t} = \left(\frac{\mathcal{A}_n}{\gamma_0 \bar{a}_n}\right)^{\frac{(1+\gamma_1)^t - 1}{\gamma_1 (1+\gamma_1)^t}},$$

$$\Pi_{T+t-1,T+t} = \frac{1}{\alpha \bar{a}_n}\left(\frac{K_{T+t}}{L_{T+t}}\right)^{1-\alpha}.$$

(i) *These values constitute a post-bubble equilibrium path if and only if for all* $t \geq T+1$:

$$K_t > \left(\alpha \bar{a}_n \left(\frac{\Pi_{t-2,t-1}}{\Pi^*}\right)^{\zeta} \Pi^*\right)^{\frac{1}{1-\alpha}}.$$

(ii) *On this equilibrium path, the economy experiences involuntary unemployment:* $L_{T+t} < 1$ *for all* $t > 0$, *and the economy converges to the bad bubbleless steady state with involuntary unemployment and below-target inflation described in Section VA.*

PROOF:
Appendix A.A5. ∎

Figure 3 plots a simulated equilibrium path in a manner similar to the simulation in Figure 1 (the dashed horizontal lines represent the good bubbleless steady

[16] In reality, there can be shocks (not modeled here) that pull the economy out of the liquidity trap, such as a good technology shock or another bubbly episode.

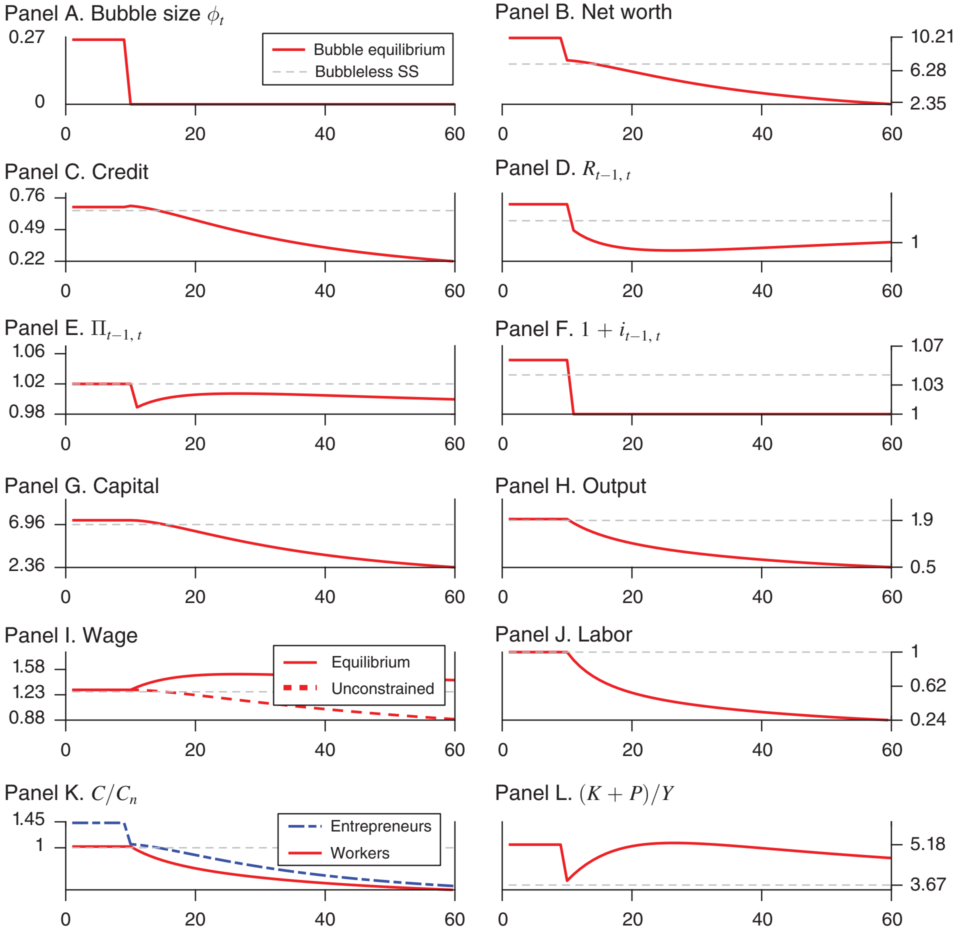

Figure 3. Persistent Post-bubble Liquidity Trap

Note: Solid lines represent detrended equilibrium variable values; gray dashed horizontal lines represent the corresponding good bubbleless steady-state values.

state).[17] As seen in the figure, the collapse causes the real and nominal interest rate to fall sharply, and the nominal interest rate hits the ZLB. The economy gradually converges to the *bad* bubbleless steady state. Intuitively, when the monetary authority is constrained by the ZLB, inflation is below the target. Low inflation exacerbates the DNWR, causing more unemployment. Higher unemployment in turn further reduces the marginal product of capital and the interest rates, creating a vicious cycle that perpetuates the liquidity trap.

[17] Again the simulation is for a model with exogenous TFP growth rate g and partial capital depreciation rate δ. Parameter values are $\gamma_0 = 0.98$, $\gamma_1 = 0.015$, $\Pi^* = 1.02$, and $\zeta = 1.5$ (following Schmitt-Grohé and Uribe 2017), while the rest are as in Section IIIB.

VI. Conclusion

We have developed a tractable rational bubbles model with downward wage rigidity. We show that expansionary bubbles could boost economic activities, but their collapse can push the economy into a persistent slump with involuntary unemployment, and investment, output, and consumption depressed below the pre-bubble levels. Under certain conditions, the economy is better off without stochastic bubbles altogether. The model's predictions are consistent with stylized features of recent bubbly episodes. The model highlights the trade-off between the economic gains during the boom due to the bubble and the loss from the bust. A macroprudential leaning-against-the-bubble policy of taxing speculative investment can help balance this boom-bust trade-off.

The model has several limitations. For instance, the model predicts that even though stochastic bubbles can reduce welfare, a perfectly safe bubble is desirable, as it helps mitigate financial frictions without any of the downside risk of an inefficient slump. Specifically, there is nothing in our model to inherently prevent a bubble from sustaining forever. Thus, the model cannot address the concern of policymakers that some rapid increases in asset prices are unsustainable. Incorporating elements from models with information friction may help address this issue (see Brunnermeier and Oehmke 2013 or Barlevy 2018 for a survey). The model also features no equilibrium default and hence cannot address the fact that corporate and household bankruptcy rates rose sharply after the collapse of the Japanese or US housing bubble. This drawback can potentially be addressed by incorporating an agency problem (e.g., Allen, Barlevy, and Gale 2017; Bengui and Phan 2018) into our framework. We leave these as potential avenues for future research.

Appendix: Omitted Proofs

A1. *Proof of Proposition 4*

PROOF:

For workers, define $\Delta W \equiv W_b - W_n$. From (46) and (47):

$$\Delta W = \frac{\log\left((1-\alpha)\,K_b^{\alpha}\right)}{1-\rho\beta} - \frac{\log\left((1-\alpha)K_n^{\alpha}\right)}{1-\beta} + \frac{\beta(1-\rho)}{1-\rho\beta}\left(\Gamma_0(s^*) + \Gamma_1(s^*)\log K_b\right).$$

Recall from (45) that $\lim_{\gamma\to 1} s^* = \infty$. Thus,

$$\lim_{\gamma\to 1}\Gamma_0(s^*) = \frac{\log(1-\alpha)}{1-\beta} + \frac{\beta(1-\alpha)}{(1-\beta)^2}\log K_n,$$

$$\lim_{\gamma\to 1}\Gamma_1(s^*) = \frac{\alpha-\beta}{(1-\beta)^2},$$

and

$$\lim_{\gamma \to 1} \Delta W = \frac{\log\left((1-\alpha)K_b^\alpha\right)}{1-\rho\beta} - \frac{\log\left((1-\alpha)\,K_n^\alpha\right)}{1-\beta}$$

$$+ \frac{\beta(1-\rho)}{1-\rho\beta}\left(\frac{1}{1-\beta}\log(1-\alpha) + \frac{\beta(1-\alpha)}{(1-\beta)^2}\log K_n + \frac{\alpha-\beta}{(1-\beta)^2}\log K_b\right)$$

$$= \underbrace{\frac{\alpha(1-\beta)^2 + \beta(1-\rho)(\alpha-\beta)}{(1-\rho\beta)(1-\beta)^2}}_{<0 \text{ if } \rho<\bar{\rho}} \underbrace{(\log K_b - \log K_n)}_{>0}.$$

If the bubble is sufficiently risky, so that

$$\rho < \bar{\rho} \equiv 1 - \frac{\alpha\,(1-\beta)^2}{\beta(\beta-\alpha)},$$

then the ratio above is negative, implying $\lim_{\gamma\to 1} \Delta W < 0$. Thus, there exists $\gamma_w < 1$ such that if $\gamma > \gamma_1$ and $\rho < \bar{\rho}$, then $W_b < W_n$.

For entrepreneurs, we similarly define $\Delta V \equiv V_b - V_n$. From (A6) and (A9) of the online Appendix and by taking $\gamma \to 1$, we can derive the following limit:

$$\lim_{\gamma \to 1} \Delta V = G(\phi), \tag{A1}$$

where $G(\phi)$ is a decreasing function with $G(0) = 0$:

$$G(\phi) \equiv \frac{\beta(\beta\phi+\theta)\left((1-\rho)\log\left(\frac{\theta(1-\phi)}{\beta\phi+\theta}\right) + \rho\log\left(\frac{\theta\rho(1-\phi)}{\theta\rho - \phi(\beta(1-\rho)+\theta)}\right)\right)}{(1-\beta)(1-\beta\rho)(\beta+\theta)}$$

$$- \frac{\beta^2\phi\left(\log\left(\frac{\beta+\theta}{\beta}\right) + \frac{1}{\sigma}\right)}{(1-\beta)(1-\beta\rho)(\beta+\theta)}$$

$$- \frac{\left((2-\alpha)\beta - 1\right)\log\left(\frac{1}{1-\beta\phi}\right)}{(1-\alpha)(\beta-1)^2} + \frac{\frac{(\sigma-1)(\alpha-\beta)}{\alpha-1} + \frac{(\beta-1)\beta}{\beta\rho-1}}{(\beta-1)^2\sigma}\log\left(\frac{1}{1-\phi}\right)$$

$$+ \frac{\beta}{(1-\beta\rho)(1-\beta)}\int_{\bar{a}_b}\log\left(\frac{\beta}{(\beta+\theta)(1-\theta)}\left(1 - \frac{\bar{a}_b}{a}\theta\right)\right)dF(a)$$

$$- \frac{\beta}{(1-\beta\rho)(1-\beta)}\int_{\bar{a}_n}\log\left(\frac{\beta}{(\beta+\theta)(1-\theta)}\left(1 - \frac{\bar{a}_n}{a}\theta\right)\right)dF(a).$$

Since in equilibrium $\phi > 0$, it follows that $\lim_{\gamma \to 1} \Delta V < 0$. Thus, there exists $\gamma_e < 1$ such that if $\gamma > \gamma_e$, then $V_b < V_n$. The proof is complete by letting $\bar{\gamma} = \max\{\gamma_w, \gamma_e\}$. ■

A2. *Proof of Corollary 6*

PROOF:

First, we show that there exists $\gamma_e < 1$ such that if $\gamma > \gamma_e$, then $V_b(\tau) \leq V_n$ for all $\tau \leq \bar{\tau}$, with equality if and only if $\tau = \bar{\tau}$. Fix any $\tau < \bar{\tau}$. By applying the same algebraic manipulations as in Section A1, we have $\lim_{\gamma \to 1} V_b(\tau) - V_n = \tilde{G}(\phi(\tau))$, where $\tilde{G}$ is a decreasing function of $\phi(\tau)$ with $\tilde{G}(0) = 0$:

$$\tilde{G}(\phi(\tau))$$

$$\equiv \frac{\beta(\beta\phi(\tau)+\theta)\left((1-\rho)\log\left(\frac{\theta(1-\phi(\tau))}{\beta\phi(\tau)+\theta}\right)+\rho\log\left(\frac{(1-\tau)\theta(1-\phi(\tau))+\tau\phi(\tau)(\beta+\theta)\frac{\theta(1-\phi(\tau))}{\theta+\beta\phi(\tau)}}{(1-\tau)\rho\theta-(1-\tau\rho)\phi(\tau)(\beta(1-(1-\tau)\rho)+\theta)}\rho\right)\right)}{(1-\beta)(1-\beta\rho)(\beta+\theta)}$$

$$-\frac{\beta^2\phi\left(\log\left(\frac{\beta+\theta}{\beta}\right)+\frac{1}{\sigma}\right)}{(1-\beta)(1-\beta\rho)(\beta+\theta)}$$

$$-\frac{((2-\alpha)\beta-1)\log\left(\frac{1}{1-\beta\phi(\tau)}\right)}{(1-\alpha)(\beta-1)^2}+\frac{\frac{(\sigma-1)(\alpha-\beta)}{\alpha-1}+\frac{(\beta-1)\beta}{\beta\rho-1}}{(\beta-1)^2\sigma}\log\left(\frac{1}{1-\phi(\tau)}\right)$$

$$+\frac{\beta}{(1-\beta\rho)(1-\beta)}\int_{\bar{a}_b(\phi(\tau))}\log\left(\frac{\beta}{(\beta+\theta)(1-\theta)}\left(1-\frac{\bar{a}_b(\phi(\tau))}{a}\theta\right)\right)dF(a)$$

$$-\frac{\beta}{(1-\beta\rho)(1-\beta)}\int_{\bar{a}_n}\log\left(\frac{\beta}{(\beta+\theta)(1-\theta)}\left(1-\frac{\bar{a}_n}{a}\theta\right)\right)dF(a)$$

Since in equilibrium $\phi(\tau) > 0$, it follows that $\lim_{\gamma\to1} V_b(\tau) - V_n = \tilde{G}(\phi(\tau)) < 0$. Thus, for all $\epsilon > 0$, there exists $\gamma(\epsilon) < 1$ such that $V_b(\tau) - V_n < \tilde{G}(\phi(\tau)) + \epsilon$ whenever $\gamma > \gamma(\epsilon)$. By letting $\epsilon = -\tilde{G}(\phi(\tau))$ and $\gamma_e = \gamma(-\tilde{G}(\phi(\tau)))$, we then get $V_b(\tau) < V_n$ for all $\gamma > \gamma_e$, as desired. Finally, note that when $\tau = \bar{\tau}$, the bubble disappears, i.e., $\phi(\bar{\tau}) = 0$, and thus $V_b(\bar{\tau}) = V_n$.

Similarly, we show that there exists $\gamma_w < 1$ such that if $\gamma > \gamma_w$ and $\rho < \bar{\rho}$, then $W_b(\tau) \leq W_n$ for all $\tau \leq \bar{\tau}$, with equality if and only if $\tau = \bar{\tau}$. Fix any $\tau < \bar{\tau}$. If $K_b(\tau) \leq K_n$, i.e., the taxed bubble is contractionary, then it is obvious that the welfare of workers cannot be better off with the bubble than without, as the equilibrium wage in the bubbly equilibrium path would always be smaller than w_n—the wage in the bubbleless steady state. Thus, we can focus on the expansionary case of $K_b(\tau) > K_n$. By applying the same algebraic manipulations as in Section A1, we get

$$\lim_{\gamma \to 1} W_b(\tau) - W_n = H(\tau) \equiv \frac{\alpha(1-\beta)^2 + \beta(1-\rho)(\alpha-\beta)}{(1-\rho\beta)(1-\beta)^2}\left(\log K_b(\tau) - \log K_n\right),$$

where the right-hand side $H(\tau)$ is strictly negative whenever $\rho < \bar{\rho}$. Thus, for all $\epsilon > 0$, there exists $\gamma(\epsilon)$ such that $W_b(\tau) - W_n < H(\tau) + \epsilon$. By letting $\epsilon = -H(\tau)$ and $\gamma_w = \gamma(-H(\tau))$, we then get $W_b(\tau) < W_n$ for all $\gamma > \gamma_w$, as desired. Finally, when $\tau = \bar{\tau}$, the bubble disappears and thus trivially $W_b(\bar{\tau}) = W_n$.

The proof is complete by letting $\bar{\gamma} = \max\{\gamma_w, \gamma_e\}$, as it is immediate from the results above that $\arg\max_{\tau \leq \bar{\tau}} \lambda W_b(\tau) + (1-\lambda)V_b(\tau) = \bar{\tau}$ when $\gamma > \bar{\gamma}$ and $\rho < \bar{\rho}$. ∎

A3. *Proof of Proposition 7*

First, we show inflation is at target and employment is full in the period the bubble collapses.

LEMMA 10: $\Pi_{T-1,T} = \Pi^*$ *and* $L_T = 1$.

PROOF:

Recall the economy is still in the bubbly steady state in $T-1$, and therefore the nominal interest rate $i_{T-1,T}$ is determined by the unconstrained Taylor rule $1 + i_{T-1,T} = R_b \Pi^*$. Also recall the Fisher equation that equates the expected return from nominal bond holding and real lending for entrepreneurs below the threshold $\bar{a}_b$:

$$1 + i_{T-1,T} = \frac{\rho u'(c_b^L) R_b \Pi^* + (1-\rho)u'(c_T^L) R_{T-1,T}\Pi_{T-1,T}}{\rho u'(c_b^L) + (1-\rho)u'(c_T^L)},$$

where the superscript L denotes entrepreneurs with productivity below $\bar{a}_b$. Here, we have used the fact that in the good state that the bubble persists in period T (which happens with probability ρ), the economy continues to be in the bubbly steady state with consumption level c_b^L for the L-type, the real interest rate is R_b, and inflation is Π^*. The indifference condition above implies:

$$R_b \Pi^* = R_{T-1,T}\Pi_{T-1,T}.$$

In addition, recall that the real interest rate between $T-1$ and T is given by

$$R_{T-1,T} = \bar{a}_b \alpha \left(\frac{L_T}{K_T}\right)^{1-\alpha} = R_b L_T^{1-\alpha}.$$

Thus the equation above reduces to

$$\Pi^* = \Pi_{T-1,T} L_T^{1-\alpha}. \tag{A2}$$

Now suppose on the contrary that $L_T < 1$. Then the DNWR must bind at T:

$$\frac{w_T}{w_{T-1}} = \frac{\gamma(L_T)}{\Pi_{T-1,T}}.$$

By substituting the first-order condition of firms (7), we then get

$$L_T^{-\alpha} = \frac{\gamma(L_T)}{\Pi_{T-1,T}}. \tag{A3}$$

Equations (A2) and (A3) then imply

$$\Pi^* = \gamma_0 L_T^{1+\gamma_1} < \gamma_0.$$

However, this violates assumption (55).

Therefore, it must be that $L_T = 1$. Equation (A2) then implies $\Pi_{T-1,T} = 1$. ∎

Now the proof for the proposition follows straightforwardly.

PROOF OF PROPOSITION 7:

Given $\Pi_{T-1,T} = \Pi^*$, it is immediate that (56) is equivalent to

$$R^f_{T,T+1}\Pi^* < 1.$$

As the bubble has collapsed in $T+1$, the real interest rate with full employment is simply given by

$$R^f_{T,T+1} = \bar{a}_n \alpha K_{T+1}^{\alpha-1},$$

as the post-bubble economy follows the bubbleless dynamics. In addition, from the law of motion of capital, we have $K_{T+1} = \alpha \mathcal{A}_n K_T^\alpha L_T^{1-\alpha} = \alpha \mathcal{A}_n K_b^\alpha$. Therefore, $R^f_{T,T+1}\Pi^* < 1$ if and only if $\bar{a}_n \alpha (\alpha \mathcal{A}_n K_b^\alpha)^{\alpha-1} < 1/\Pi^*$, which is equivalent to (56). ∎

A4. *Proof of Lemma 8*

PROOF:

Suppose on the contrary that $i_{t-1,t} = 0$ but $L_t = 1$. The DNWR constraint is slack, implying that $w_t^f/w_{t-1} \geq \gamma_0/\Pi_{t-1,t}$, or equivalently, inflation must be sufficiently high:

$$\frac{K_t^\alpha}{\left(K_{t-1}/L_{t-1}\right)^\alpha} \geq \frac{\gamma_0}{\Pi_{t-1,t}}. \tag{A4}$$

However, the inflation rate is determined by the Fisher equation:

$$1 = \underbrace{\bar{a}_n \alpha K_t^{\alpha-1}}_{R_{t-1,t} \text{ with } L_t=1} \Pi_{t-1,t}. \tag{A5}$$

Substitute (A5) into (A4), we get a condition that the capital stock must be sufficiently high:

$$K_t \geq \gamma_0 \bar{a}_n \alpha \left(K_{t-1}/L_{t-1}\right)^{\alpha}.$$

Substituting the law of motion of capital $K_t = \mathcal{A}_n \alpha K_{t-1}^{\alpha} L_{t-1}^{1-\alpha}$ yields

$$L_{t-1} \geq \frac{\gamma_0 a^L}{\mathcal{A}_n}.$$

However, as $1 \geq L_{t-1}$, it then follows that $1 \geq \gamma_0 a^L / \mathcal{A}_n$, which contradicts assumption (55). ∎

A5. *Proof of Proposition 9*

PROOF:

For notation simplicity, let us normalize the period when the bubble bursts to be period 0; that is, $T = 0$. Then, on the unemployment path $L_1, L_2, \ldots < 1$ and $i_{0,1} = i_{1,2} = \cdots = 0$. The unemployment path can be characterized as follows. The flow of capital is given by

$$K_t = \mathcal{A}_n \alpha K_{t-1}^{\alpha} L_{t-1}^{1-\alpha}.$$

Binding DNWR and ZLB provide the following two equations, respectively,

$$\Pi_{t-1,t} \frac{\left(K_t/L_t\right)^{\alpha}}{\left(K_{t-1}/L_{t-1}\right)^{\alpha}} = \gamma\left(1 - L_t\right),$$

$$\underbrace{\bar{a}_n \alpha K_t^{\alpha-1} L_t^{1-\alpha}}_{R_{t-1,t}} \Pi_{t-1,t} = 1.$$

Combining the two above equations yields

$$\frac{\left(K_t/L_t\right)^{\alpha}}{\left(K_{t-1}/L_{t-1}\right)^{\alpha}} = \bar{a}_n \alpha K_t^{\alpha-1} L_t^{1-\alpha} \gamma\left(1 - L_t\right).$$

Rewriting the above equation by utilizing the parameterization of $\gamma(\cdot)$,

$$K_t \left(\frac{L_{t-1}}{K_{t-1}}\right)^{\alpha} = \bar{a}_n \alpha \gamma_0 L_t^{1+\gamma_1}.$$

By substituting in the flow of capital, we find a recursive form for labor:

$$L_t = \left(\frac{\mathcal{A}_n}{\gamma_0 \bar{a}_n} L_{t-1}\right)^{\frac{1}{1+\gamma_1}}.$$

Similarly, inflation can be expressed as a function of last period's labor and capital:

$$\begin{aligned}\Pi_{t-1,t} &= \frac{1}{R_{t-1,t}} = \frac{1}{\bar{a}_n \alpha}\left(\frac{K_t}{L_t}\right)^{1-\alpha} \\ &= \frac{1}{a^L \alpha}\left(\gamma_0 \bar{a}_n \alpha K_{t-1}^{\alpha} L_{t-1}^{\frac{\gamma_1-\alpha(1+\gamma_1)}{1+\gamma_1}}\left(\frac{\mathcal{A}_n}{\gamma_0 \bar{a}_n}\right)^{\frac{\gamma_1}{1+\gamma_1}}\right)^{1-\alpha}.\end{aligned}$$

These expressions can be further simplified by recursively plugging in for $L_{t-1}, L_{t-2}, \ldots, L_1$. Therefore, labor, L_t, can be written as a function of $L_0 = 1$ (as shown in Appendix A.A3, there is full employment in the period the bubble bursts) and t:

$$\begin{aligned}L_t &= \left(\frac{\mathcal{A}_n}{\gamma_0 \bar{a}_n} L_{t-1}\right)^{\frac{1}{1+\gamma_1}} \\ &= \left(\left(\frac{\mathcal{A}_n}{\gamma_0 \bar{a}_n}\right)^{\sum_{s=0}^{t-1}\left(\frac{1}{1+\gamma_1}\right)^s} \underbrace{L_0^{\left(\frac{1}{1+\gamma_1}\right)^{t-1}}}_{=1}\right)^{\frac{1}{1+\gamma_1}} \\ &= \left(\frac{\mathcal{A}_n}{\gamma_0 \bar{a}_n}\right)^{\frac{(1+\gamma_1)^t-1}{\gamma_1(1+\gamma_1)^t}}.\end{aligned}$$

Similarly, using the flow of capital equation and working backward, K_t can be written as a function of K_0, t, and all past L_t:

$$\begin{aligned}K_t &= \mathcal{A}_n \alpha K_{t-1}^{\alpha} L_{t-1}^{1-\alpha} \\ &= (\mathcal{A}_n \alpha)^{\sum_{s=0}^{t-1}\alpha^s} K_0^{\alpha^t}\left(\prod_{s=1}^{t-1} L_{t-s}^{\alpha^{s-1}}\right)^{1-\alpha} \\ &= (\mathcal{A}_n \alpha)^{\frac{1-\alpha^t}{1-\alpha}} K_0^{\alpha^t}\left(\frac{\mathcal{A}_n}{\gamma_0 \bar{a}_n}\right)^{\frac{1-\alpha}{\gamma_1}\left(\frac{1-\alpha^{t-1}}{(1-\alpha)} - \frac{1-(\alpha(1+\gamma_1))^{t-1}}{(1-\alpha(1+\gamma_1))(1+\gamma_1)^{t-1}}\right)}.\end{aligned}$$

For these values to constitute an equilibrium path after the collapse of the bubble in period T, the necessary and sufficient conditions are that the DNWR and the ZLB do indeed bind. From Proposition 8, we know it is sufficient to show that the ZLB binds, i.e., $R^f_{t-1,t}(\Pi_{t-2,t-1})^{\zeta}(\Pi^*)^{1-\zeta} < 1$ for all t, where the real interest rate with full employment is given by $R^f_{t-1,t} = \bar{a}_n \alpha K_t^{\alpha-1}$. This inequality holds if and only if $K_t > \left(\bar{a}_n \alpha \left(\Pi_{t-2,t-1}/\Pi^*\right)^{\zeta}\Pi^*\right)^{1/(1-\alpha)}$ for all t, as stated in the proposition.

Finally, it is algebraically straightforward to show that $\lim_{t\to\infty} K_{T+t} = K$, $\lim_{t\to\infty} L_{T+t} = L$, and $\lim_{t\to\infty} \Pi_{T+t-1,T+t} = \Pi$, where K, L, and Π are the capital, labor, and inflation in the bad bubbleless steady state as established in Section II. ∎

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